Scientific Computing Lecture Series
Introduction to MATLAB Programming

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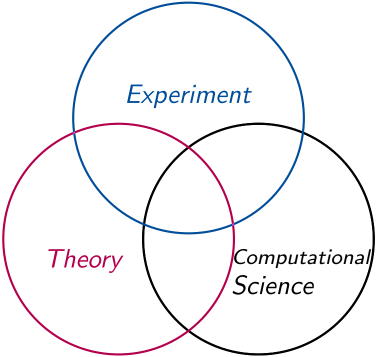
Lecture I
Basic Commands, Arrays and Matrices, Functions

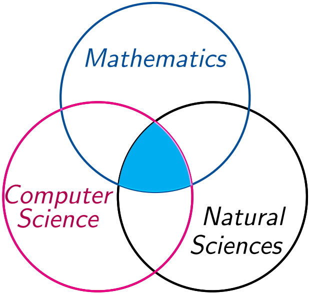
Computational Science



Computational Science now constitutes what many call the **third pillar** of the scientific enterprise, a peer alongside theory and physical experimentation.

Report to the President:"Computational Science : Ensuring America's Competitiveness", June 2005.

Scientific Computing



Scientific Computing

= Computational Science

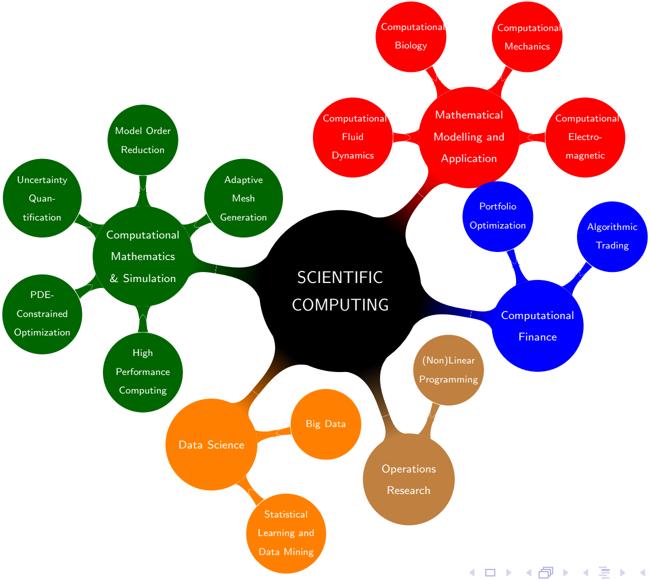
= Computational Science and Engineering

= Scientific Computation

= Computational Mathematics

[0] Quoted by https://en.wikipedia.org/wiki/Computational_science

Scientific Computing Program



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Lecture Information

MATLAB Lecture Series is organized by members of **Scientific Computing Program** of IAM:

- **February 17**: M. Alp Üreten
  - Basic Commands, Syntax, Arrays, and Matrices.
- **February 18**: Eda Oktay
  - Scripts, Functions, Control Loops, Advanced Data Structures.
- **February 19**: Sıtkı Can Toraman
  - Graphics, Visualizations, and Symbolic Toolbox.

Lecture I–Outline

1. Introduction to MATLAB
2. Data Structures: Arrays and Matrices
3. Operators
4. Sparse Matrices

What is MATLAB ?

- Matlab is a high–level language and interactive environment that enables you to perform computationally intensive tasks. It was originally designed for solving linear algebra type problems using matrices. It's name is derived from MATrix LABoratory.

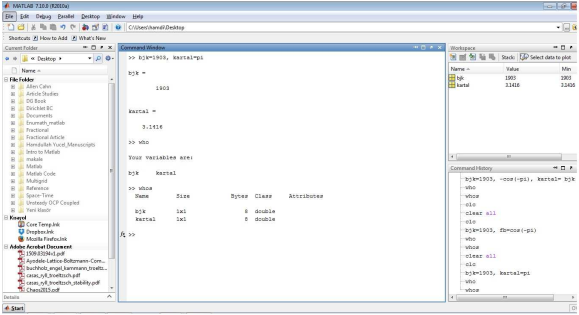
MATLAB System

- Desktop Tools and Development Environment
  - Includes the MATLAB desktop and Command Window, an editor and debugger, a code analyzer, browsers for viewing help, the workspace, files, and other tools.
- Mathematical Function Library
  - Vast collection of computational algorithms ranging from elementary functions, like sine, cosine, and complex arithmetic, to more sophisticated functions like matrix inverse, matrix eigenvalues, Bessel functions, and fast Fourier transforms.
- Language
  - The MATLAB language is a high-level matrix/array language with control flow statements, functions, data structures, input/output, and object-oriented programming features.

Continue...

- Graphics
  - MATLAB has extensive facilities for displaying vectors and matrices as graphs, as well as editing and printing these graphs. It also includes functions that allow you to customize the appearance of graphics as well as build complete graphical user interfaces on your MATLAB applications.
- External Interfaces
  - The external interfaces library allows you to write C and Fortran programs that interact with MATLAB.

MATLAB Interface



- Command Window: Here you can give MATLAB commands typed at the prompt, >>.
- Current Directory: Directory where MATLAB looks for files.
- Workspace: Shows what variable names are currently defined and some info about their contents.
- Command History: History of your commands.

Helps/Docs

- MATLAB is huge! - there is no way to remember everything you will need to know.
  - help command - shows in the Commmand Window all the ways in which you can use the command.
  - doc command - brings up more extensive help in a separate window.
  - lookfor command- searches for the keyword.

```
>> help sin
   SIN    Sine of argument in radians.
   SIN(X) is the sine of the elements of X.
   See also asin, sind.
   Overloaded methods:
                  codistributed/sin
   Reference page in Help browser
   doc sin
```

Basic Commands

- MATLAB records in the workspace and command history everything you write in the command window, so:
  - clear variable
    - deletes variable from memory (and workspace)
  - clear all
    - deletes all variables from memory (and workspace)
  - clc
    - cleans command window
  - save
    - save variables to a file (.mat format)
  - load
    - load variable bindings into the environment (look at workspace, the variables a is back)
- MATLAB's command window works like a Linux terminal
  - cd, mkdir, rmdir, ls, . . .

Basic Commands

- Some commands used to interact with MATLAB
  - what
    - returns the MATLAB files (.m , .mat) in the current directory
  - who
    - returns the variables in your workspace
  - whos
    - returns the variables in the workspace with additional info (size, dimensions)
  - Try typing why in the command window. You will see that MATLAB is also a Philosopher!

Variables

- MATLAB is a weakly typed language
  - No need to initialize variables!
- Just assign some value to a variable name, and MATLAB will automagically understand its type
  - $x = 3$ double
  - $x =$ 'hello' char
- MATLAB supports various types, the most often used are
  - 64-bit double (default)
  - 16-bit char
- Most variables you will deal with will be vectors or matrices of doubles or chars
- Other types are also supported: complex, symbolic, 16-bit and 8-bit integers, etc.

Variables

- Naming Conventions
  - Have not to be previously declared
  - Variable names can contain up to 63 characters
  - To create a variable, simply assign a value to a name

```
>>  var1 = 1903;
>>  myStrings = 'merhaba';
```

  - Variable names
    - first character must be LETTER
    - after that, any combination of letters, numbers and _
    - allowable: NetCost, Left2Pay, X3, BJK1903
    - not allowable: Net-Cost, 1903BJK, %x, @sign
  - Variable names are case sensitive (var1 is different from Var1)

Variables

- Avoid to use built–in variables such as
  - ans — Default variable name for results
  - eps — Smallest incremental number
  - pi — Value of π
  - inf — Infinity
  - NaN — Not a number e.g. 0/0
  - i and j — represent complex numbers

Scalars

- A variable can be given a value explicitly (shows up in workspace!)

```
>>  a = 1903
```

- Or as a function of explicit values and existing variables

```
>>  c = 2.4*24-4*a
```

- To suppress output, and the line with a semicolon ;

```
>>  h = 22/7;
```

Arrays

- Like other programming languages, arrays are an important part of MATLAB
- Two types of arrays
  - matrix of numbers (either double or complex)
  - cell array of objects (more advanced data structure)
- Row vector: comma or space separated values between brackets

```
>> row = [1 4 6 7]
>> row = [1,4,6,7]
```

- Column vector: semicolon separated values between brackets

```
>> column = [1.4;2;pi]
```

- Size of a vector: length

```
>> l = length(column)
```

Special Vector Constructors

- linspace()

```
>> a = linspace(0,10,5)

a =

     0    2.5000    5.0000    7.5000   10.0000
```

- Colon operator (:). The basic syntax is

inital:stepsize:final

```
>> m = 3:8, r = 0:0.25:1, s=1:-1

m =

     3     4     5     6     7     8

r =

     0    0.2500    0.5000    0.7500    1.0000

s =

   Empty matrix: 1-by-0
```

- logspace (to initialize logarithmically spaced values)

Matrices

- Make matrices like vectors

```
>> A = [5 7 9; 1 -3 -7];
```

- Concatenation of vectors

```
>> r1 = [2 4];
>> r2 = [3 6];
>> M  = [r1; r2];
```

- Concatenation of vectors and matrices. Dimensions and Type must coincide!

```
>> r1 = [2 4];
>> m1 = [3 6; 8 12];
>> M  = [r1; m1];
```

- Getting size of the matrix

```
>> [r,c] = size(M);     % size in each dimension
>> r  = size(M,1);      c = size(M,2);
>> nd = ndims(M);       % number of dimensions
```

Special Matrices

- zeros(m,n) $m \times n$ matrix of zeros
- ones(m,n) $m \times n$ matrix of ones
- eye(n) $n \times n$ identity matrix
- rand(m,n)
  - $m \times n$ matrix of uniformly distributed random numbers in range [0,1]

```
>> M = rand(2,3)
   M = 0.8147    0.1270    0.6324
       0.9058    0.9134    0.0975
```

- randn(m,n)
  - $m \times n$ matrix of normally distributed random numbers (mean 0, std. dev. 1))

```
>> M = randn(2,3)
   M =   -0.4336    3.5784   -1.3499
          0.3426    2.7694    3.0349
```

Replicating and Concatenating Matrices

- repmat

```
>> X = [1 2;3 4];
>> Y = repmat(X,2,3)
   Y =  1     2     1     2     1     2
        3     4     3     4     3     4
        1     2     1     2     1     2
        3     4     3     4     3     4
```

- vertcat

```
>> v1 = [2 3 4];   v2 = [1 2 3];
>> X = vertcat(v1,v2)
   X =  2     3     4
        1     2     3
```

- horzcat

```
>> v1 = [2; 3; 4];   v2 = [1; 2; 3];
>> X = horzcat(v1,v2)
   X = 2  1
       3  2
       4  3
```

Reshaping Matrices

- Using the : operator

```
>> x = round(10*rand(2,4));
>> y = x(:);  (The elements of x are stacked in a
               column vector, column after column)
```

- reshape()

```
>> x2 = reshape(y,2,4);
>> M = reshape(linspace(11,18,8),[2,2,2])
   M(:,:,1) =
             11    13
             12    14
   M(:,:,2) =
             15    17
             16    18
```

Vector Indexing

- MATLAB indexing starts with 1, not 0
- a(n) returns n^{th} element
- The index argument can be vector. In this case, each element is looked up individually, and returned as a vector of the same size as the index vector.

```
>> x = [4 6 7 -1  0];
>> a = x(2:4); --------> a=[6 7 -1];
>> b = x(1:end-2); ----> b=[4 6  7];
```

Matrix Indexing

- using subscripts (row and column)

```
>> A = [1:3;4:6;7:9];
>> A(1:2,:)
ans =
     1     2     3
     4     5     6
>> A([3 1], [2 3])
ans =
     8     9
     2     3
>> A([1:2],:) = []   % delete row 1 and 2
A =
   7     8     9
```

- using linear indices (as if matrix is vector)

```
>> [A(2), A(4), A(9)]
ans =
     4     2     9
```

Matrix Indexing

- To select rows and columns of a matrix

```
>> c = [1 4; 0 2];
>> d = c(1,:)
   d =
       1     4
```

- To get the min. (or max.) value and its index

```
>> a = [ 1 -1 0 -4, 21];
>> [minVal,minInd] = min(a)
minVal =  -4  minInd =   4
```

- To find any indices of specific values or ranges

```
>> ind = find(a==0);
>> ind = find(a > 0 & a < 4);
```

- To convert between subscripts and indices, use ind2sub and sub2ind

Operations

- Arithmetic operations (+,-,*,/)

```
>> 7/45
>> (2+i)*4/5
```

- Exponentiation (^)

```
>> (3+2*j)^2
```

- Complicated expressions, use parentheses

```
>> ((2+3)*3)^0.5
```

- Multiplication is NOT implicit given parenthesis

```
>> 3(1+0.7)
??? 3(1+0.7)
    |
Error: Unbalanced or unexpected parenthesis or bracket.
```

- MATLAB has an enormous library of built-in-functions

```
>> sqrt(2), log(2), log(10)(0.23), cos(pi), atan(2.5)
>> exp(1903), round(1.4),  floor(3.3), ceil(4.23)
```

Transpose

- The transpose operators turns a column vector into a row vector and vice versa
- The $'$ gives the Hermitian-transpose, i.e., transposes and conjugates all complex numbers
- For vectors of real numbers $.'$ and $'$ give same result

```
>> a = [ 1;5; 3i+2]
>> a'
ans =
   1.0000     5.0000    2.0000 - 3.0000i
>> transpose(a)
ans =
   1.0000     5.0000    2.0000 + 3.0000i
>> a.'
ans =
   1.0000     5.0000    2.0000 + 3.0000i
```

Element-Wise Functions

- All functions that work on scalars also works on vectors

```
>> t = [1, pi, 0];
>> f = exp(t);
>> f = [exp(1) exp(pi) exp(0)];
```

- To do element-wise operations, use the dot: $.*, ./, .\hat{}$. Both dimensions must match (unless one is scalar)

```
>> u=1:2:8, v=u.^2, w=u./v
   u =
      1     3     5     7
   v =
      1     9    25    49
   w =
      1.0000    0.3333    0.2000    0.1429
>> A = [ 5 7 9; 1 -3 -7];  B = [-1 2 5; 9  0  5];
>> A.*B
   ans =
       -5    14    45
        9     0   -35
```

Rational and Logical Operators

- Boolean values: zero is false, nonzero is true
- Some of the logical operators:

Operator	Meaning
$<, <=, >, >=$	less than, less than or equal to, etc.
$==, \sim=$	equal to, not equal to
&	logical AND
$\vert$	logical OR
$\sim$	logical NOT
all	all true
any	any true

Logical Indexing

- Construct a matrix R

```
>> R = rand(5)
   R =
       0.8147    0.0975    0.1576    0.1419    0.6557
       0.9058    0.2785    0.9706    0.4218    0.0357
       0.1270    0.5469    0.9572    0.9157    0.8491
       0.9134    0.9575    0.4854    0.7922    0.9340
       0.6324    0.9649    0.8003    0.9595    0.6787
```

- Test for some logical cases

```
>> R(R<0.15)'
  ans =
        0.1270    0.0975    0.1419    0.0357
>> isequal(R(R<0.15), R(find(R<0.15)))
  ans =
        1
```

Find

- find returns indices of nonzero values. It can simplify code and help avoid loops
- basic syntax: index = find(condition)

```
>> x = rand(1,10)

   x =

      Columns 1 through 5

      0.4505    0.0838    0.2290    0.9133    0.1524

      Columns 6 through 10

      0.8258    0.5383    0.9961    0.0782    0.4427

>> inds = find(x>0.4 & x<0.7)

   inds =

          1      7     10

>> x(inds)

   ans =

         0.4505    0.5383    0.4427
```

Dense Matrices

- Dense matrix is a matrix in which most of its elements are nonzero.
- Any classical approach to create a matrix results a dense matrix in MATLAB.

`[,]`	creates a single row matrix
`[;]`	creates a singe column matrix
`zeros(n)`	returns an $n \times n$ matrix of zeros
`ones(n)`	returns an $n \times n$ matrix of 1s
`diag()`	creates diagonal matrix of given vector

- Create a 1000×1000 matrix A

$$\begin{bmatrix} -2 & 1 & & & & \\ 1 & -2 & 1 & & & \\ & \ddots & \ddots & \ddots & & \\ & & & 1 & -2 & 1 \\ & & & & 1 & -2 \end{bmatrix}$$

```
M = 1000;
A = diag(ones(M-1,1),-1) + diag(-2*ones(M,1),0) + diag(ones(M-1,1),1);
```

- Compute how much storage this dense matrix need

```
s = whos('A');
by = s.bytes;
>> by = 8000000 bytes
```

Sparse Matrices

- A sparse matrix is a matrix which has relatively small number of nonzero elements.
- Triplet Format in MATLAB stores values and their corresponding row and column values.

```
row = [1 2 3 1 5 4 1 5];
col = [1 1 2 3 3 4 5 5];
val = [2 8 9 2 4 5 7 3];

S = sparse(row,col,val);
```

$$S = \begin{bmatrix} 2 & 0 & 2 & 0 & 7 \\ 8 & 0 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 4 & 0 & 3 \end{bmatrix}$$

- spalloc() creates an all zero allocation for a sparse matrix.

```
m = 10;        % number of rows
n = 10;        % number of columns
nz = 21;       % number of nonzero entries
S = spalloc(m,n,nz);
```

- spones() generates a matrix of 1s with same sparsity structure as matrix S

```
M = spones(S);
```

- speye() constructs a sparse identity matrix of size $m \times n$

```
I = speye(m,n);
```

- spdiags() extracts or constructs sparse diagonal matrices.
    - Extracts nonzero diagonal entries from matrix S

      ```
      B = spdiags(S);
      ```

    - Extracts diagonals of S specified by d

      ```
      B = spdiags(S,d);
      ```

    - Replaces the diagonals of S specified by d with columns of B

      ```
      S = spdiags(B,d,S);
      ```

    - Create $m \times n$ sparse matrix from the columns of B and place them along the diagonals specified by d

      ```
      S = spdiags(B,d,m,n)
      ```

- Create a 1000×1000 matrix S

$$\begin{bmatrix} -2 & 1 & & & & \\ 1 & -2 & 1 & & & \\ & \ddots & \ddots & \ddots & & \\ & & & 1 & -2 & 1 \\ & & & & 1 & -2 \end{bmatrix}$$

```
M = 1000;
S = spdiags([ones(M,1) -2*ones(M,1) ones(M,1)], [-1 0  1] , M, M);
```

- Compute how much storage this dense matrix need

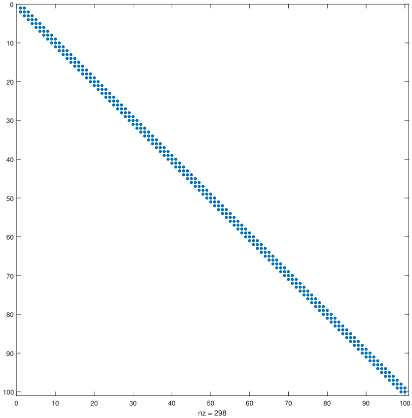
```
s = whos('S');
by = s.bytes;
>> by = 55976 bytes
```

- full() converts a sparse matrix to a dense matrix

```
A = full(S)
```

- spy() plots sparsity structure of a matrix.

```
spy(S)
```



- Do not change sparsity structure
- Indexing in a sparse structures is a expensive procedure
  - Accessing the row and column indexes i, j and changing previous value $S(i,j) = c$ is required
- Accessing values is slow in sparse matrices
  - When an element $S(i,j)$ is requested, a search trough row and column values is needed

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